A General Purpose Computer Program for the Dynamic Simulation of Vehicle-Guideway Interactions

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A general purpose computer program for calculating the dynamic response of vehicles traveling over surface or evaluated guideways is described. The program has application to a broad class of transportation systems and hence eliminates the need for numerous specialized programs. The program is modular in design and is based on the finite element or building block method in which a complex dynamic system is made up of a number of components. The equations of motion for each of these components is known, and the program automatically combines the component equations to form equations of motion for the complete system. The equations of motion are then integrated numerically to give the response of the system to variables such as guideway roughness, span length, etc. Several output options are available and on-line printer plots or off-line CalComp plots of the response can be obtained. Addition building blocks can be easily added to the program whenever desired. The program is written in PL/I language and has been used on the IBM 360/91 computer. The program has been used on a limited number of problems, several of which are included in this paper.

I. Introduction

A GENERAL purpose computer program (APLDYN)[1] has been written for the simulation of the dynamics associated with vehicles traveling over surface or elevated guideways with prescribed irregularities. The program has application to a broad class of transportation systems and hence eliminates the need for numerous specialized programs. It is based on the finite element method in which a complex system is made up of a number of discrete components. This technique is not new to the structural engineer, but it has seen limited application to dynamic problems associated with discrete moving masses. The APLDYN application of this finite element technique to moving vehicle problems is similar to, but more general than that used by Hunt[2] to determine the dynamic response of a flexible vehicle moving along a flexible supporting structure.

Other simulations[3-5] although useful for studying the dynamic behavior of specific systems, have been limited in scope and versatility. Kaplan et al.[3] have developed a simulation for the vertical response of a single car or a two-car train traveling across an elevated guideway with no irregularities. The guideway flexibility provides the dynamic excitation. McHenry and Deleys[4] have written a simulation for the response of automobiles when encountering obstacles such as guard rails, railroad crossings and abutments. The simulation does not handle the more general class of vehicles, elevated guideways, and roadbed irregularities encountered in new transportation systems. Melpar has also developed a comprehensive computer program[5] but this program is written specifically for a conventional railcar on a surface track. The Melpar program handles a multidegree-of-freedom system but is limited in the number of mass elements that can be simulated.

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The APLDYN program can be used in screening new system concepts, performing design tradeoff studies of system components, comparing alternative approaches, and resolving specific dynamic problems that may arise in the prototype development. The simulation considers translational and rotational motions in three orthogonal directions, single and multicar trains, and linear and nonlinear suspension systems.

The program is written in PL/I language and has been used on the IBM 360/91 computer. The maximum problem size is limited only by the size of the available computer. Several output options exist and the user can request as little or as much output as needed. On-line printer plots or off-line CalComp plots of the response can be obtained. In this paper a general description of the program, its basic dynamic theory, and a few example problems are presented.

II. General Description of Program

In APLDYN the system is made up of nodes which define the degrees of freedom; elements or building blocks which consist of springs, dampers, masses, beams, or guideway spans; forces which are externally applied forces; and constraints which define the relationships between the dependent and independent degrees of freedom. Constraints include many functions such as defining the guideway irregularity and positioning the vehicle on the guideway. New elements or building blocks can be added to the program for application to new or different systems. The equations of motion for each of the components is known and the program automatically combines the component equations to form the equations of motion of the complete system. The equations of motion are then integrated numerically to give the response of the system to variables such as guideway roughness, span characteristics, suspension characteristics, vehicle speed, etc.

APLDYN is capable of solving any dynamic response problem which can be modeled from rigid masses and flexible beams which are connected by linear or nonlinear springs and dampers, subjected to periodic and aperiodic forcing functions, and constrained by linear variable constraints. Most transportation vehicles can be modeled by a set of rigid masses connected by springs and dampers, and most elevated guideways can be modeled as simply supported beams. Constraint conditions provide a connection between the vehicle and its guideway and define the guideway roughness. The guideway roughness data are read from a special input file which may be a data set stored in the computer system. This feature avoids reading in roughness data from cards for each run.

The program has a library of equations of motion for a group of finite elements which it combines into the equations of motion for the entire system. The finite element library can easily be extended to include equations of motion for additional elements if desired.

At present the user has a choice of six integration routines: 1) Euler's method, 2) trapezoid rule, 3) Runge-Kutta method, 4) predictor-corrector method, 5) Newmark's beta method, and 6) Nordsieck's method.

Methods 1 and 2 are extremely simple and were used in the development of the program; the other methods are more sophisticated. Although the Runge-Kutta method is relatively slow, it is very stable and has been used quite successfully. The Newmark's beta method is slightly more efficient than the Runge-Kutta and has also proven to be quite useful. The predictor-corrector and the Nordsieck's (also a predictor-corrector) methods should be very efficient since they require only two evaluations of the derivative for each time step; however, some difficulties have been encountered when applying these to moving vehicles on elevated guideways.

The user can control the amount of output. For debugging a model, a user may find it helpful to print out all quantities at first, and after his model is corrected, print out only those quantities of interest. The output format has been made as readable as possible, in the hope that it will be self explanatory. As output options, the user can call for a printout of all forces, displacements, velocities, and accelerations, or he can print and make printer plots of only selected variables. Output data may be stored for use in making CalComp plots.

The program has been designed with emphasis on ease in preparing data. All data can be in a free form format, hence the user need not count columns on the input cards. In most cases if a data item does not apply to a specific case it need not be supplied as data. As input the program requires the following information: 1) title, 2) list of indices—defines amount of data to be read in, 3) node data—definition of the degrees of freedom of the system, 4) element data—description of the various components of the system and how they are interconnected, 5) external force data—description of external applied forces acting on the system, 6) guideway roughness data—a table of guideway roughness data, 7) constraint data—definition of linear constraint conditions and specified displacements, 8) list of program options, 9) initial condition data—values to begin integration, and 10) data for output options—several of the output options require data.

The program is constructed in a modular plan thus making the program easy to modify. The following is a list of the program modules: 1) main—calls other routines as required, 2) element assembly routine, 3) linear element library, 4) nonlinear element library, 5) integration routines, 6) output routine, 7) routine to sum forces, 8) external force routine, 9) constraint routine, and 10) guideway roughness routine.

The program has been designed to accept additions and modifications with minimum difficulty. The computer run time depends on many factors including the number of degrees of freedom, the number of time steps, the amount of output requested, the integration routine used, and the number of nonlinear elements. The ratio of the computer run time to simulation time is about 1 for very simple problems and about 30 for a problem with 24 degrees of freedom.

III. Analytical Formulation

The finite element method is one of the most widely used methods in structural analysis. In this method the system under study is divided into a number of elements which are connected together at nodal points. This method has the advantage of versatility (with suitable elements, a wide range of problems can be solved) and ease of automation (the method readily lends itself to automatic computation). The present discussion of the finite element method is intended to provide a summary of the theory as it is used in the computer program.

Equations of Motion

The equations of motion of a dynamic system may be derived using Lagrange's equation

$$(d/dt)(\partial T/\partial \dot{x}_i) - \partial T/\partial x_i + \partial \mathscr{F}/\partial \dot{x}_i + \partial V/\partial x_i = X_i \tag{1}$$

where

T = kinetic energy
$\mathscr{F}$ = Rayleigh's dissipation function
V = potential energy
x_i = generalized coordinates (degrees of freedom)
X_i = components of generalized force

Equation (1) is quite general, and may be applied to a system as a whole or to just one element of a system to obtain the equations of motion. The inertia effects of the masses are included using kinetic energy, the effects of linear spring forces are included in the potential energy, and the effects of linear damping forces are included in the Rayleigh function. Provisions have been made in the program for both linear and nonlinear forces. External forces and forces of nonlinear elements are included in the X_i.

For linear systems the kinetic energy, potential energy, and dissipation function may be written

$$T = \tfrac{1}{2}\dot{Y}^T M \dot{Y}, \quad \mathscr{F} = \tfrac{1}{2}\dot{Y}^T D \dot{Y}, \quad V = \tfrac{1}{2}Y^T K Y \tag{2}$$

where

Y = displacement vector of the system
M = mass matrix
D = damping matrix
K = stiffness matrix

Applying Eq. (1) to Eqs. (2) results in the equation

$$M\ddot{Y} + D\dot{Y} + KY = F(t) + G(t, Y, \dot{Y}) \tag{3}$$

where

$F(t)$ = external forcing functions
$G(t, Y, \dot{Y})$ = nonlinear forces

Rewriting Eq. (3) in the following form

$$M\ddot{Y} = F(t) + G(t, Y, \dot{Y}) - D\dot{Y} - KY \tag{4}$$

gives the familiar $F = ma$ formula in matrix notation. For a linear element the equation of motion may be written as

$$m_i \ddot{y}_i + d_i \dot{y}_i + k_i y_i = f_i(t) \tag{5}$$

where

y_i = element displacement vector
m_i = element mass matrix
d_i = element damping matrix
k_i = element stiffness matrix
$f_i(t)$ = external forcing function on element

Figure 1 shows a general structure divided into elements. The elements are connected together at what are often called nodes. The nodes are the degrees of freedom of the element and/or system.

For each element there is a connection matrix (C_i) such that

$$y_i = C_i Y \tag{6}$$

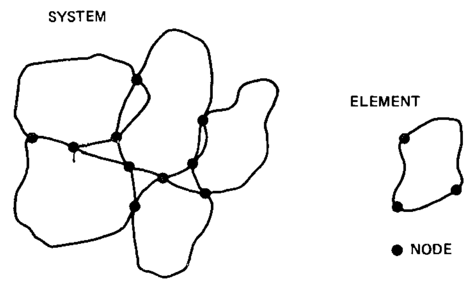

Fig. 1 General arrangement of a system, its elements, and its nodes.

These matrices contain information on how the elements are connected together to form the system. The elements of matrix C_i will be zeros and ones. For a linear system, the kinetic energy of the system is the total of the kinetic energy of the elements. A similar relationship holds for the dissipation function and potential energy

$$T = \Sigma \tfrac{1}{2} \dot{y}_i{}^T m_i \dot{y}_i, \quad \mathscr{F} = \Sigma \tfrac{1}{2} \dot{y}_i{}^T d_i \dot{y}_i, \quad V = \Sigma \tfrac{1}{2} y_i{}^T k_i y_i \qquad (7)$$

Using Eq. (1), the equation of motion of the system becomes

$$M\ddot{Y} + D\dot{Y} + KY = F(t) \qquad (8)$$

where

$$M = \Sigma C_i{}^T m_i C_i, \quad D = \Sigma C_i{}^T d_i C_i, \quad K = \Sigma C_i{}^T k_i C_i \qquad (9)$$

Equations (9) show how the equations of motion of the entire system are assembled from those of the elements. This process involves only matrix algebra and is easily adopted to automatic computation.

Equation (8) contains only linear forces. To include the nonlinear components, let $g_i(t, y_i, \dot{y}_i)$ be the nonlinear force of the ith element. The total force on the system due to all the nonlinear elements is therefore

$$G(t, Y, \dot{Y}) = \Sigma C_i{}^T g_i(t, C_i Y, C_i \dot{Y}) \qquad (10)$$

The equation of motion of a system of linear and nonlinear elements is given by Eq. (4) where the element assembly process is given by Eqs. (9) and (10).

Constraint Relations

The degrees of freedom in the vector Y are not always independent. This often is the case in the analysis of vehicles traveling on flexible guideways. To solve such problems, constraint relations must be supplied. A constraint equation of the form

$$Y = CY_c + Y_0 \qquad (11)$$

is considered where the quantities are functions of time and defined as

$Y_0(t)$ = specified displacements
$Y_c(t)$ = vector of independent degrees of freedom
$C(t)$ = constraint matrix
$Y(t)$ = vector of independent and dependent degrees of freedom

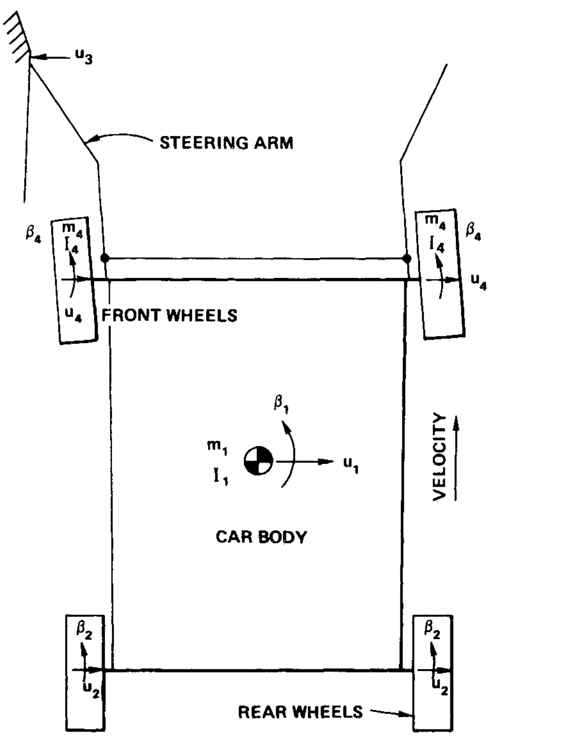

Fig. 2 Model for problem 1, lateral response of a rubber-tired vehicle to a transient excitation.

Guideway roughness is specified in $Y_0(t)$ while the effects associated with positioning vehicles on elevated guideways are specified in $C(t)$. Combining Eqs. (11) and (3) results in the equation

$$C^TMC\ddot{Y}_c + 2C^TM\dot{C}\dot{Y}_c + C^TM\ddot{C}Y_c + C^TDC\dot{Y}_c + C^TD\dot{C}Y_c + C^TKCY_c = C^TF + C^TG - C^TM\ddot{Y}_0 - C^TD\dot{Y}_0 - C^TKY_0 \qquad (12)$$

For the special case of a time independent constraining matrix (C) Eq. (12) reduces to

$$C^TMC\ddot{Y}_c + C^TDC\dot{Y}_c + C^TKCY_c = C^TF + C^TG - C^TM\ddot{Y}_0 - C^TD\dot{Y}_0 - C^TKY_0 \qquad (13)$$

Either Eq. (12) or (13) is used by the program. The vector Y_c is obtained by numerical integration of either Eq. (12) or (13), and Y is then calculated from Eq. (11).

Incorporated in the constraint routine are four functions which provide guideway roughness

1) $y(t) = C_1 \sin(C_2 t + C_3)$
2) $z(x) = C_1 \sin(C_2 x + C_3)$
3) $y(t) = f(t)$
4) $z(x) = g(x)$

Two of these are functions of time and two of distance. Since the vehicle travels at constant velocity we have

$$x = vt$$

so that

$$y(t) = z(vt) = g(vt) = g(x)$$

Values of the latter two functions are determined by linear interpolation from a table of values.

Node Description

Nodes are the degrees of freedom (DOF) of the system. Each node may have one or more components and the components may be rectilinear displacements, rotations, or generalized displacements. The program contains eleven different node types thereby permitting the user to solve a wide variety of problems. The user need only use the exact number of DOF needed, no extra components need to be constrained out. Thus, a wide range of problems from simple to complex can be solved efficiently.

Element Library

The program contains a number of specific elements, thereby permitting a variety of systems to be modeled. For complete list and derivation of elements see Ref. 1. The elements which are provided fall into the following categories: rigid body masses, gyroscopic effects, tire side-slip, springs and dampers, and bending and torsion of rods.

IV. Example Problems

Three problems were selected to illustrate several features of the program. One problem involves the lateral response of a vehicle to a transient displacement of its steering mechanism, one simulates the motion of a vehicle on an elevated guideway, and one studies the pitch induced roll motion of a flywheel powered automobile.

Problem 1: Response of a Rubber-Tired Vehicle to a Transient Lateral Excitation

In this problem, the program is used to study the lateral transient response of a small rubber-tired vehicle. The lateral guidance of the vehicle is provided by a wall follower which is linked to the steering mechanism of the front wheels. A lateral ramp function disturbance is applied to the vehicle and a damped transient response is observed. The degrees of freedom of interest in this problem are the lateral displacement and the yaw rotation. The tires are modeled using a side-slip element.

The dynamic model of the system is shown in Fig. 2. Values of the parameters used in the simulation are shown in Table 1.

The problem demonstrates the use of a specified displacement constraint. In this case the wall displacement is set equal to a

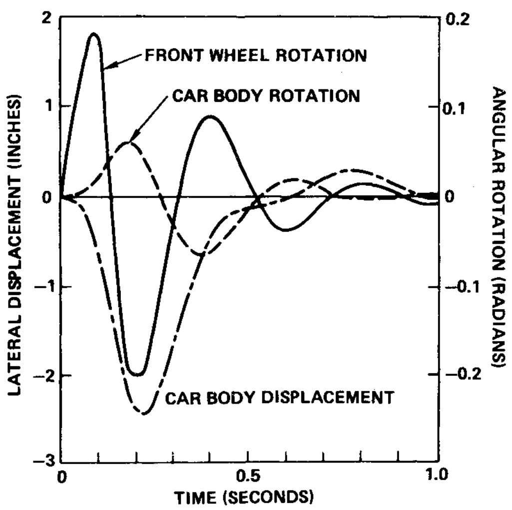

Fig. 3 Response of rubber tired vehicle to transient excitation.

Table 2 Simulation parameters used in problem 2

Masses			
Car body	m_1	16.4	lb-sec^2/in.
	I_x	13200	lb-in-sec^2
	I_y	33300	lb-in.-sec^2
Front wheels (each)	m_3, m_4	0.428	lb-sec^2/in.
Rear axle and wheels	m_2	1.36	lb-sec^2/in.
	I_x	660	lb-in-sec^2
Springs			
Tires	k	1500	lb/in.
	c	0.08	lb-sec/in.
Front suspension	k	220	lb/in.
	c	15.0	lb-sec/in.
Rear suspension	k	300	lb/in.
	c	17.5	lb-sec/in.
Torsion spring	k	5×10^5	in.-lb/rad
	c	100	in.-lb-sec/rad
Guideway	M/L	0.065	lb-sec^2/in.^{2}
	EI	1.9×10^{11}	lb-in.^{2}
	L	840	in.
	Zeta	0.02	Dimensionless

table of values which is input to the program. Also, a linear constant constraint is used to relate the rotaton of the front wheels to the displacement of the car, the rotation of the car, and the displacement of the guideway wall.

The model requires 4 nodes: three nodes containing displacement and rotation to represent the car body and the front and rear wheel sets and one node containing displacement only to represent the wall displacement. Four elements are required for this system. The car body and the front wheels are simple mass elements, while the side-slip qualities of the wheels are represented by two side-slip elements. The mass of the rear wheels is included in the car body element.

Some of the results of this problem are shown in Fig. 3. Here the rotation and displacement of the car body and the rotation of the front wheels are plotted with time. A most significant feature of the results is the rapid damping of the disturbance thereby demonstrating stability of the vehicle to transient lateral excitations.

Table 1 Parameter values used in problem 1

Side-slipe rear element	C_p	17,200 lb
Front wheels	m_4	1.00 lb-sec^2/in.
	I_4	1.00 lb-in-sec^2
Side-slip front element	C_p	17,200 lb
Car body	m_1	9.0674 lb-sec^2/in.
	I_1	8160.6 lb-in-sec^2

Problem 2: Vehicle on Elevated Guideway

This problem is again a small rubber-tired vehicle which travels on an elevated guideway. The guideway surface is considered perfectly smooth and the exciting force is the deflection of the guideway under the vehicle. Each guideway span consists of two parallel independent beams. In this problem only the heave, roll, and pitch motions of the car are of interest. The vehicle is composed of a car body, a split axle in the front, and a solid axle in the rear. A strong torsion spring is used to limit the relative motion of the car body and the independent front axles. The dynamic model of this system is shown in Fig. 4. The simulation parameters used in this problem are shown in Table 2.

In this case, the stiffness of the tires was assumed to be linear near the operating point. The damping constants of the front and rear suspensions were selected to be approximately one fourth of the critical damping, defined in the heave mode as

$$c_{cr} = (4km)^{1/2}$$

A guideway span consists of two independent beams, and the values of the guideway parameters as defined above are for each beam. Rotation of the guideway is caused only by differential bending of the beams. Although the vehicle is modeled with a roll degree of freedom, this roll motion is not excited due to the symmetrical loading.

The torsion spring that limits the relative motions of both the car body and the independent front axles is easily handled using a fictitious node and a constraint condition which relates the rotation of the node to the relative deflections of the front

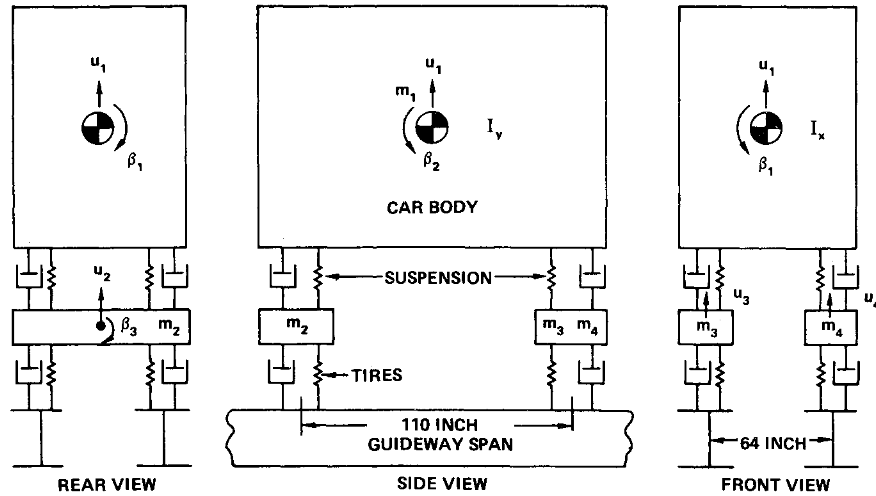

Fig. 4 Model for problem 2, vehicle on an elevated guideway.

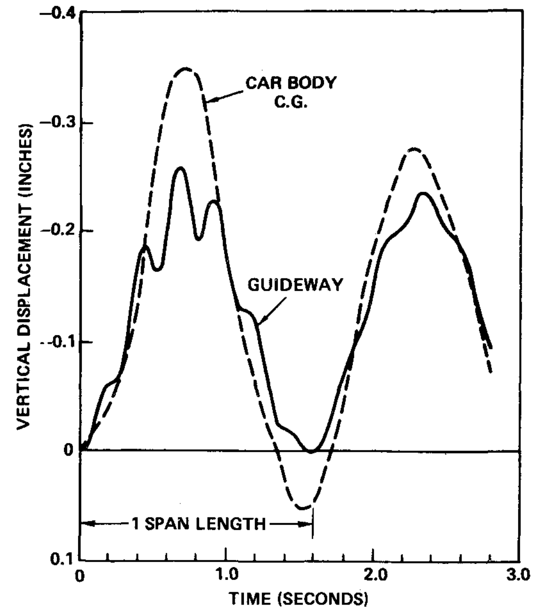

Fig. 5 Response of vehicle on an elevated guideway.

axles. The torsion spring is then connected between this node and the roll of the car body.

The initial conditions used to start the simulation represent the deflections in the masses and springs due to gravity. These initial condition cards were obtained from an earlier run wherein the car was allowed to settle on its springs with no exciting forces and zero velocity. Using the program's punch option, the final values of all variables were punched on cards and used to start the problem from the equilibrium position.

This problem demonstrates the use of the linear variable constraint. The nodes on the guideway are constrained to have the deflection of the guideway at the location of the node.

The model requires 11 nodes: 4 nodes are constrained to be on the guideway, 4 nodes at masses, 1 fictitious node represents the rotation of the independent front axles, and 2 nodes for the two guideway spans. The model also requires 15 elements: 4 masses, 4 linear springs for the tires, 4 linear springs for the suspension, 1 torsion spring, and 2 guideway elements. Five constraint conditions are used: 4 linear variable constraints to position the car on the guideway, and 1 linear constant constraint linking the rotational node to the differential deflection of the front axles.

In Fig. 5 are shown the vertical displacement of the car body center of gravity and the vertical displacement of the guideway under the rear tires. The results illustrate the smoothing action of the vehicle's suspension system. Output data in the form of acceleration and jerk histories could also be obtained.

Problem 3: Transient Response of a Flywheel-Powered Family Car Encountering a Bump

In this problem APLDYN is used to study the gyroscopic

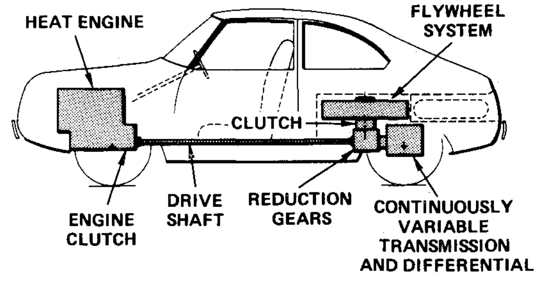

Fig. 6 Schematic diagram of flywheel powered automobile.

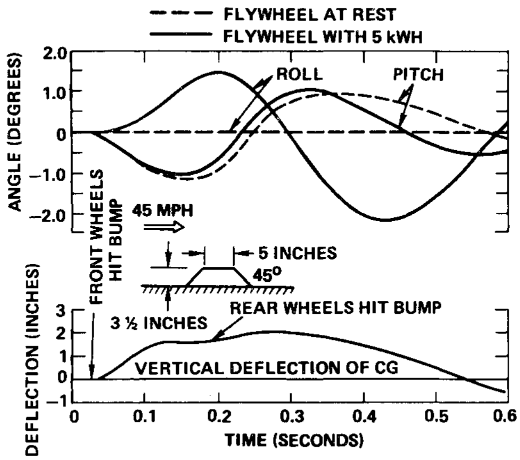

Fig. 7 Response of family car encountering a severe roadway obstacle at 45 mph with 5 kwh hard-mounted flywheel.

effects of an automobile which is powered by a high energy flywheel (Fig. 6). Of interest are the pitch and roll motions of the automobile as it encounters a severe roadway obstacle. The model used is similar to that of Problem 2 with the addition of the flywheel element. In this case data for a full size American family car is used along with a 5 kw-hr flywheel (164 lb rotor at 30,000 rpm). The flywheel is assumed to be hard mounted to the car body.

The vehicle was programed over a $3\frac{1}{2}$-in.-high bump in the roadway while traveling at 45 mph. The bump profile and the resulting displacement are shown in Fig. 7. In the upper graph the dashed lines are for a nonrotating flywheel, and it is seen that there is no roll, induced by the bump. With the flywheel running at 30,000 rpm the pitching of the vehicle induces a roll component nearly twice as large as the pitch. The vertical deflections are the same for both cases.

V. Concluding Remarks

The APLDYN program has been effectively applied to evaluate the dynamics associated with several different types of transportation systems. In this respect, the general purpose feature of the program has been demonstrated.

Although the current version of the program contains a large selection of elements or building blocks, some restrictions exist as to the type of problem that can be handled; i.e., flanged wheel-rail elements are not presently included. The addition of new elements into the program is provided for and the incorporation of these new elements should be relatively straightforward.

Use of the program is also currently restricted to straight track (no curves). Modification of the program to accommodate curved track would not be difficult.

Referemces

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